CORRELATION THEORY OF PRECRITICAL DEFORMATIONS OF THIN ELASTIC SHELLS

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As is known [1 and 2], the deformations of thin elastic loaded shells are quite sensitive to small initial deviations of the middle surface from the ideal shape. In particular, this is manifest in the large spread in experimental data in stability tests on shells. Stochastic problems in shell theory are customarily solved by applying direct methods. The distributed system is hence replaced by a system, equivalent in some sense, with a finite number of degrees of freedom. Approximate solutions of this kind leave a feeling of dissatisfaction. Moreover, if we start from the linearized equations, then under some sufficiently broad assumptions, exact solutions of the stochastic boundary value problem are obtained successfully [3].

The problem is solved below on the basis of equations obtained by linearizing the shell theory equations in the neighborhood of the initial state of stress. An additional assumption on the smallness of the scale of the initial deviations and of the scale of their correlation as compared to the characteristic dimensions of the middle surface is used, as is also an assumption on the homogeneity of the field of initial deviations. General formulas are deduced for the correlation functions, the variance and the spectral densities of the parameters of the stress-strain state of the shell. The results are expressed in terms of tabulated functions for a broad class of isotropic initial deviations. This permits a study of the dependence of the correlation properties of the displacements, deformations and stresses on the properties of the initial deviations and on the initial stresses in the middle surface.

1. Let us consider a thin elastic shell with initial deviations from the ideal shape. Let the external loading be such that a pure membrane state of stress originates in the ideal shell, the buckling modes are rapidly varying functions of the coordinates, and the critical parameters depend negligibly slightly on the shell dimensions and the boundary conditions on its outline. Moreover, let the deviations from the ideal shape be sufficiently small, and have sufficiently small scales of variability and correlation. For loadings not to close to the critical, displacements of points of the middle surface of a loaded shell will possess these properties. Nonlinear shell theory equations of Mushtari-Vlasov type [4] shall be used in the form:

$$D\Delta\Delta w - s^{\alpha\lambda}s^{\beta\mu}(b_{\alpha\beta} + \nabla_\alpha\nabla_\beta w)\nabla_\lambda\nabla_\mu\chi = p$$
$$(1/Eh)\,\Delta\Delta\chi + s^{\alpha\lambda}s^{\beta\mu}\left(b_{\alpha\beta} + \tfrac{1}{2}\nabla_\alpha\nabla_\beta w\right)\nabla_\lambda\nabla_\mu w = 0 \qquad (1.1)$$

Here $w(x^1, x^2)$ is a function of the normal displacements, $\chi(x^1, x^2)$ is a function of the tangential stress resultants, D is the cylindrical stiffness, E the elastic modulus, h the shell thickness, p the normal loading intensity, $b^{\alpha\beta}$a tensor of the initial curvature of the middle surface, $s^{\alpha\beta}$ the unit antisymmetric tensor on the middle surface. If the deviations from the ideal surface are slight, we can then put

$$b_{\alpha\beta} = b^{(0)}{}_{\alpha\beta} + \varepsilon b^{(1)}{}_{\alpha\beta} \tag{1.2}$$

where $b^{(0)}_{\alpha\beta}$ is the curvature tensor of the ideal tensor, ε is a small parameter. Neglecting the change in metric because of membrane deformation, we seek the solution of (1.1) as

$$w = \varepsilon w_1 + \varepsilon^2 w_2 + \ldots, \qquad \chi = \chi_0 + \varepsilon\chi_1 + \varepsilon^2\chi_2 + \ldots \tag{1.3}$$

Substitution of (1.2) and (1.3) yields after comparing terms containing ε,

$$D\Delta\Delta w_1 - s^{\alpha\lambda}s^{\beta\mu}b_{\alpha\beta}{}^{(0)}\nabla_\lambda\nabla_\mu\chi_1 - N^{\alpha\beta}\nabla_\alpha\nabla_\beta w_1 = N^{\alpha\beta}\nabla_\alpha\nabla_\beta w_0$$

$$(1\,/\,Eh)\,\Delta\Delta\chi_1 + s^{\alpha\lambda}s^{\beta\mu}b_{\alpha\beta}{}^{(0)}\nabla_\lambda\nabla_\mu w_1 = 0 \tag{1.4}$$

It has here been taken into account that $s^{\alpha\lambda}\ s^{\beta\mu}\ b^{(0)}_{\alpha\beta}\ \Delta_\lambda\ \Delta_\mu\ \chi_0 = p$. Moreover, the notation $N^{\alpha\beta}$ has been introduced for the tensor of the initial membrane stress resultants, and the correction to the curvature tensor (1.2) has been expressed in terms of the function of the initial deviations $w_0(x^1, x^2)$

$$s^{\alpha\lambda}s^{\beta\mu}\nabla_\lambda\nabla_\mu\chi_0 = N^{\alpha\beta}, \qquad b_{\alpha\beta}{}^{(1)} = \nabla_\alpha\nabla_\beta w_0$$

Since, by assumption, the scales of variation and correlation of the functions w_0, w_1 and χ_1 are small as compared with the scales of variation of the metric properties of an ideal middle surface, (1.4) can be simplified by being rewritten in orthogonal coordinates (the lines of curvature) $\mathbf{r} = x_1, x_2$ with unit metric tensor, and by replacement of the tensor derivatives by the corresponding partial derivatives

$$\begin{gathered} D\Delta\Delta w_1 - \left(\frac{1}{R_2}\frac{\partial^2\chi_1}{\partial x_1^2} + \frac{1}{R_1}\frac{\partial^2\chi_1}{\partial x_2^2}\right) - N_{\alpha\beta}\frac{\partial^2 w_1}{\partial x_\alpha \partial x_\beta} = N_{\alpha\beta}\frac{\partial^2 w_0}{\partial x_\alpha\,\partial x_\beta} \\ \frac{1}{Eh}\,\Delta\Delta\chi_1 + \left(\frac{1}{R_2}\frac{\partial^2 w_1}{\partial x_1^2} + \frac{1}{R_1}\frac{\partial^2 w_1}{\partial x_2^2}\right) = 0 \end{gathered} \tag{1.5}$$

Here R_1, R_2 are the principal radii of curvature of the ideal middle surface; the rule of summation over the subscripts α, β has been retained.

2. Let $w_0(\mathbf{r})$ be a random function of the coordinates, with mathematical expectation zero. Let us consider a domain sufficiently remote from the boundaries and other lines of distortion. It can be expected that under the assumptions made above on the nature of the loading, and for a rapidly changing field of initial deviations $w_0(\mathbf{r})$ the influence of the boundaries on the shell behavior in the inner domain will be sufficiently small. Then the influence of the boundaries can generally be neglected by replacing the boundary conditions by the requirement of boundedness of the functions at infinity. If the shell parameters and the initial membrane stress resultants can be assumed constant on a sufficiently large domainoof the middle surface, and the function of the initial imperfections $w_0(\mathbf{r})$ can be considered as a homogeneous random field, then the stochastic problem is solved by a well known method [3].

A homogeneous random field $\phi(\mathbf{r})$ admits of spectral representation as a stochastic Fourier-Stieltjes integral

$$\varphi(\mathbf{r}) = \int e^{i\mathbf{kr}}\,dZ(\mathbf{k}) \qquad (\mathbf{k} = k_1, k_2) \tag{2.1}$$

The distribution function $Z(\mathbf{k})$ satisfies the relationship

$$\langle dZ(\mathbf{k})\,dZ^*(\mathbf{k}')\rangle = \Phi(\mathbf{k})\,\delta(\mathbf{k} - \mathbf{k}')\,d\mathbf{k}\,d\mathbf{k}' \tag{2.2}$$

where the angular brackets denote the average over the set of realizations, the asterisks denote the transition to the complex conjugate, $d\mathbf{k} = dk_1\,dk_2$, $\delta(\mathbf{k})$ is the two-dimensional delta function. The function $\Phi(\mathbf{k}) \geq 0$ is the spectral density of the random field $\Phi(\mathbf{r})$. The corresponding correlation function $R(\rho)$, where $\rho = \mathbf{r} - \mathbf{r}_1$ is expressed in terms of $\Phi(\mathbf{k})$ according to the Wiener-Khinchine theorem

$$R(\rho) = \mathrm{Re}\iint\limits_{-\infty}^{\infty} \Phi(\mathbf{k})\,e^{i\mathbf{k}\rho}\,d\mathbf{k} \tag{2.3}$$

Let us represent the random fields $w_0(\mathbf{r})$, $w_1(\mathbf{r})$ and $\chi_1(\mathbf{r})$ as integrals of type (2.1), and let us take into account that the fields are interrelated by means of (1.5). Utilizing formulas of type (2.2), we obtain the following formulas for the spectral densities of the fields $w_1(\mathbf{r})$ and $\chi_1(\mathbf{r})$:

$$\Phi_{w_1}(\mathbf{k}) = F^2(\mathbf{k})\,\Phi_{w_0}(\mathbf{k}), \qquad \Phi_{\chi_1}(\mathbf{k}) = G^2(\mathbf{k})\,\Phi_{w_0}(\mathbf{k})$$

$$F(\mathbf{k}) = \frac{k_\alpha k_\beta N_{\alpha\beta}}{k^4 D + (Eh/k^4)(k_1^2/R_2 + k_2^2/R_1)^2 + k_\alpha k_\beta N_{\alpha\beta}} \qquad (2.4)$$

$$G(\mathbf{k}) = \frac{Eh}{k^4}\left(\frac{k_1^2}{R_2} + \frac{k_2^2}{R_1}\right)F(\mathbf{k}) \qquad (k^2 = k_1^2 + k_2^2)$$

The spectral densities of the remaining parameters of the stress-strain state of the shell are expressed in terms of $\Phi_{w_1}(\mathbf{k})$ and $\Phi_{\chi_1}(\mathbf{k})$, as well as in terms of their joint spectral density

$$\Phi_{w_1\chi_1}(\mathbf{k}) = F(\mathbf{k})\,G(\mathbf{k})\,\Phi_{w_0}(\mathbf{k}) \qquad (2.5)$$

Thus, the fluctuating stresses σ_{11} at the points $z = \pm h/2$ are defined as

$$\sigma_{11} = \frac{1}{h}\frac{\partial^2 \chi_1}{\partial x_2^2} \pm \frac{Eh}{2(1-\nu^2)}\left(\frac{\partial^2 w_1}{\partial x_1^2} + \nu\frac{\partial^2 w_1}{\partial x_2^2}\right)$$

Hence, taking account of (2.1) and (2.2) we obtain the spectral density (2.6)

$$\Phi_{\sigma_{11}}(\mathbf{k}) = \frac{k_1^2}{h^2}\Phi_{\chi_1}(\mathbf{k}) \pm \frac{E}{1-\nu^2}(k_1^2 + \nu k_2^2)\,\Phi_{w_1\chi_1}(\mathbf{k}) + \frac{E^2h^2}{4(1-\nu^2)}(k_1^2 + \nu k_2^2)\,\Phi_{w_1}(\mathbf{k})$$

Formulas of the type (2.4) to (2.6) permit some general deductions on the change in spectrum composition of the fields $w_1(\mathbf{r})$, $\chi_1(\mathbf{r})$ etc. as a function of the nature and magnitude of the initial membrane stress resultants. Let us examine the expression for $F(\mathbf{k})$. In substance, the function $F(\mathbf{k})$ is the transfer function of a system connecting the initial deviations of the middle surface from its ideal shape with the additional deviations $w_1(\mathbf{r})$. Formulas (2.4) to (2.6) remain meaningful although the function $F(\mathbf{k})$ has no real poles. The equation to find the poles

$$k^4 D + \frac{Eh}{k^4}\left(\frac{k_1^2}{R_2} + \frac{k_2^2}{R_1}\right) + k_\alpha k_\beta N_{\alpha\beta} = 0 \qquad (2.7)$$

agrees with the equation to find the critical stress resultants in the linear theory of shell stability. Let us recall that the case $N_{11} > 0$, $N_{22} > 0$ corresponds to tension. Therefore, the theory is applicable although the stress resultants are less than their critical values determined by linear theory.

We find the wave numbers corresponding to the most rapidly growing deviations from the condition

$$\frac{\partial F(\mathbf{k})}{\partial k_1} = \frac{\partial F(\mathbf{k})}{\partial k_2} = 0 \qquad (2.8)$$

Let the loading be given with the accuracy of the parameter p. Replacing $N_{\alpha\beta}$ by $pN_{\alpha\beta}$ in (2.7), we obtain that the critical value of the loading parameter is

$$p_*(\mathbf{k}) = -\frac{1}{k_\alpha k_\beta N_{\alpha\beta}}\left[k^4 D + \frac{Eh}{k^4}\left(\frac{k_1^2}{R_2} + \frac{k_2^2}{R_1}\right)^2\right] \qquad (2.9)$$

On the other hand, taking account of (2.9), formula (2.4) for $F(\mathbf{k})$ is written for $p < p_*(\mathbf{k})$ as

$$F(\mathbf{k}) = \frac{p}{p_*(\mathbf{k}) - p}$$

It is hence seen that the functions $p_*(\mathbf{k})$ and $F(\mathbf{k})$ take on stationary values for the identical wave numbers k_1, k_2. The transfer function $F(\mathbf{k})$ therefore takes on the maximum value for deviations coincident with the buckling modes in linear theory. Precisely these deviations grow most rapidly, although the linearized equations (1.5) remain applicable.

3. The determination of the correlation functions by means of the spectral densities

(2.4) to (2.6) reduces to the two-dimensional Fourier transform (2.3). This transformation can be carried out only by numerical methods in the general case. Meanwhile, there is a broad class of problems for which analytical calculations can be completed.

For example, let us consider a spherical shell loaded by uniform pressure. If the initial imperfections of the shell form an isotropic field, the integral in (2.3) reduces to a single integral in the "radial" wave number k. Indeed, the formulas for the correlation functions of the fields $w_1(\mathbf{r})$ and $\chi_1(\mathbf{r})$ become

$$R_{w_1}(\rho) = \mathrm{Re}\iint\limits_{-\infty}^{\infty} F^2(k)\,\Phi_{w_0}(k)\,e^{i\mathbf{k}\rho}\,d\mathbf{k}$$

$$R_{\chi_1}(\rho) = \mathrm{Re}\iint\limits_{-\infty}^{\infty} G^2(k)\,\Phi_{w_0}(k)\,e^{i\mathbf{k}\rho}\,d\mathbf{k} \qquad (3.1)$$

where $F(k)$, $G(k)$ and $\Phi_{w_0}(k)$ depend only on the modulus k. Let ξ_1, ξ_2 denote components of the vector $\rho = \mathbf{r} - \mathbf{r}_1$. Transforming to polar coordinates $\xi_1 = \rho\cos\phi$, $\xi_2 = \rho\sin\phi$, $k_1 = k\cos\theta$, $k_2 = k\sin\theta$ and using the known relationship

$$\int\limits_0^{2\pi} \cos(k\rho\cos\theta)\,d\theta = 2\pi J_0(k\rho)$$

we reduce the first of Formulas (3.1) to

$$R_{w_1}(\rho) = 2\pi\int\limits_0^{\infty} F^2(k)\,\Phi_{w_0}(k)\,J_0(k\rho)\,k\,dk \qquad (3.2)$$

The formulas for the correlation function of the other parameters as well as for the mutual correlation functions are transformed analogously.

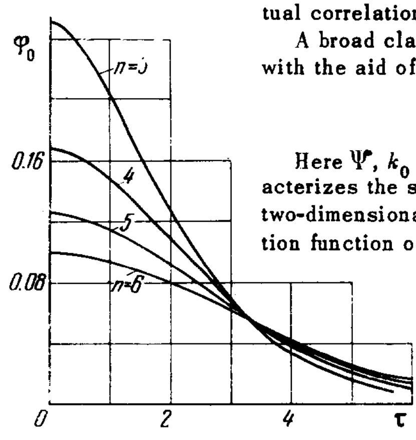

Fig. 1

A broad class of isotropic two-dimensional random fields is given with the aid of spectral densities of the form

$$\Phi_{w_0}(k) = \frac{\Psi}{(1 + k^2/k_0^2)^n} \qquad (3.3)$$

Here Ψ, k_0 and n are constants; the parameter $1/k_0$ hence characterizes the scale of correlation. The $n = 2$ case corresponds to a two-dimensional Markov field (the analog of the exponential correlation function of one independent variable). For $n > 2$ the formula describes a differentiable random field. We henceforth assume n to be an integer ($n = 3, 4, \ldots$). The correlation function of the initial deviations is defined by a formula of type (3.2)

$$R_{w_0}(\rho) = 2\pi\Psi\int\limits_0^{\infty} \frac{J_0(k\rho)\,k\,dk}{(1 + k^2/k_0^2)^n} = 2\pi k_0^2\Psi\,\frac{(k_0\rho)^{n-1}K_{n-1}(k_0\rho)}{2^{n-1}(n-1)!} \qquad (3.4)$$

where K_n is the Macdonald function of order n. A graph of the function

$$\frac{1}{2\pi k_0^2\Psi} R_{w_0} = \varphi_0(k_0\rho) = \varphi_0(\tau)$$

is shown in Fig. 1.

4. The method of contour integration [5] can be used to calculate the correlation functions for $w_1(\mathbf{r})$ and $\chi_1(\mathbf{r})$ in the case of an isotropic field of deviations with a spectral density of type (3.3).

For example, let us examine the correlation function of the deflection $w_1(r)$ in the form

(3.2). Let us write it as

$$R_{w_1} = 2\pi k_0^2 \, \Psi \, \varphi(\tau) \tag{4.1}$$

and let us evaluate the integral

$$c\varphi(\tau) = \int_0^\infty \frac{J_0(\varkappa\tau/\varkappa_0)\,\varkappa^5 d\varkappa}{(\varkappa_0^2+\varkappa^2)^n(\varkappa^4+\beta^2\varkappa^2+1)^2} \tag{4.2}$$

where

$$\varkappa = \frac{k}{k_*}, \quad \varkappa_0 = \frac{k_0}{k_*}, \quad k_* = \left(\frac{Eh}{DR^2}\right)^{1/4}, \quad c = \frac{1}{\beta^4 \varkappa_0^{2(n-1)}}$$

$$\beta^2 = \frac{N}{Dk_*^2} = \frac{NR}{Eh^2}[12(1-\nu^2)]^{1/2}, \qquad \tau = k_0\rho \tag{4.3}$$

Let us replace the integrand by the complex variable function

$$f(z) = \frac{H_0^{(1)}(z\tau/\varkappa_0)\,z^5}{(\varkappa_0^2+z^2)^n(z^4+\beta^2z^2+1)^2} \tag{4.4}$$

Here $H_0^{(1)}(z\tau/\varkappa_0)$ is the zero order Hankel function. If the initial stress resultant N is greater than the critical value, i.e., $\beta^2 > -2$, then the function $f(z)$ is holomorphic everywhere in the upper half-plane including the real axis as well, with the exception of a finite number of poles and branch points $z = 0$. Let us compute the sum of the residues around all the poles of the function $f(z)$ in the upper half-plane. Expression (4.2) becomes

$$c\varphi(\tau) = \frac{(-1)^{n-1}}{2^{n-1}(n-1)!}\left(\frac{d}{\varkappa_0 d\varkappa_0}\right)^{n-1}\left[\frac{\varkappa_0^4 K_0(\tau)}{(\varkappa_0^4-\beta^2\varkappa_0^2+1)^2}\right] -$$

$$- \frac{1}{2\gamma_1}\frac{d}{d\gamma_1}\left[\frac{\gamma_1^4 K_0(\gamma_1\tau/\varkappa_0)}{(\varkappa_0^2-\gamma_1^2)^n(\gamma_2^2-\gamma_1^2)^2}\right] - \frac{1}{2\gamma_2}\frac{d}{d\gamma_2}\left[\frac{\gamma_2^4 K_0(\gamma_2\tau/\varkappa_0)}{(\varkappa_0^2-\gamma_2^2)^n(\gamma_1^2-\gamma_2^2)^2}\right]$$

$$\gamma^2_{1,2} = 1/2\,[\beta^2 \mp (\beta^4-4)^{1/2}] \tag{4.5}$$

A particular case of (4.5) is the formula for the dimensionless correlation function of plate displacements (the formula is suitable only for tensile stresses in the middle surface)

$$c\varphi(\tau) = \frac{(-1)^{n-1}}{2^{n-1}(n-1)!}\left(\frac{d}{\varkappa_0\,d\varkappa_0}\right)^{n-1}\left[\frac{K_0(\tau)}{(\varkappa_0^2-\beta^2)^2}\right] +$$

$$+ \frac{(\varkappa_0^2-\beta^2)(\tau/\varkappa_0)K_1(\beta\tau/\varkappa_0) - 2\beta n K_0(\beta\tau/\varkappa_0)}{2\beta(\varkappa_0^2-\beta^2)^{n+1}} \tag{4.6}$$

The notation (4.3) has been retained in (4.6). The parameter R in the coefficient k_* is here the characteristic length.

5. Let us consider the case $n = 3$ in more detail. The expression for the dimensionless correlation function (4.5) becomes

$$c\varphi(\tau) = a_1\gamma_1\tau K_1(\gamma_1\tau/\varkappa_0) + a_2\gamma_2\tau K_1(\gamma_2/\tau\varkappa_0) + a_3 K_0(\gamma_1\tau/\varkappa_0) +$$
$$+ a_4 K_0(\gamma_2\tau/\varkappa_0) + (a_5 + a_6\tau^2)K_0(\tau) + a_7\tau K_1(\tau) \tag{5.1}$$

The coefficients a_j are expressed in terms of $\varkappa_0$, γ_1 and γ_2 as follows:

$$a_1 = \frac{\gamma_1^2}{2(\varkappa_0^2-\gamma_1^2)^3(\gamma_2^2-\gamma_1^2)^2\varkappa_0}; \qquad a_2 = \frac{\gamma_2^2}{2(\varkappa_0^2-\gamma_2^2)^3(\gamma_1^2-\gamma_2^2)^2\varkappa_0}$$

$$a_3 = \frac{\gamma_1^2(3\gamma_1^4-\gamma_1^2\gamma_2^2-2\varkappa_0^2\gamma_2^2)}{(\varkappa_0^2-\gamma_1^2)^4(\gamma_2^2-\gamma_1^2)^3}, \qquad a_4 = \frac{\gamma_2^2(3\gamma_2^4-\gamma_1^2\gamma_2^2-2\varkappa_0^2\gamma_1^2)}{(\varkappa_0^2-\gamma_2^2)^4(\gamma_1^2-\gamma_2^2)^3}$$

$$a_5 = \frac{3\varkappa_0^8-8\varkappa_0^4\gamma_1^2\gamma_2^2+2\varkappa_0^2\gamma_1^2\gamma_2^2(\gamma_1^2+\gamma_2^2)+\gamma_1^4\gamma_2^4}{(\varkappa_0^2-\gamma_1^2)^4(\varkappa_0^2-\gamma_1^2)^4} \tag{5.2}$$

$$a_6 = \frac{1}{8(\varkappa_0^2-\gamma_1^2)^2(\varkappa_0^2-\gamma_2^2)^2}, \qquad a_7 = \frac{5\varkappa_0^4-\varkappa_0^2(\varkappa_1^2+\gamma_2^2)-3\gamma_1^2\gamma_2^2}{4(\varkappa_0^2-\gamma_1^2)^3(\varkappa_0^2-\gamma_2^2)^3}$$

An analogous formula for a plate will be

$$c\varphi(\tau)=\frac{(\beta\tau/\varkappa_0)K_1(\beta\tau/\varkappa_0)}{2\beta^2(\varkappa_0^2-\beta^2)^3}+\frac{(5\varkappa_0^2-\beta^2)\tau K_1(\tau)}{4\varkappa_0^4(\varkappa_0^2-\beta^2)^3}+$$
$$+\frac{\tau^2K_0(\tau)}{8\varkappa_0^4(\varkappa_0^2-\beta^2)^2}+\frac{3\left[K_0(\tau)-K_0(\beta\tau/\varkappa_0)\right]}{(\varkappa_0^2-\beta^2)^4}\tag{5.3}$$

Passage to the limit in (5.1) and (5.3) as $\tau \to 0$ is of interest since we then obtain the corresponding values of the dimensionless variance. For a plate such a limit passage yields:

$$c\varphi(0)=\frac{(\varkappa_0^2-\beta^2)(2\varkappa_0^4+5\beta^2\varkappa_0^2-\beta^4)-12\varkappa_0^4\beta^2\ln(\varkappa_0/\beta)}{4\beta^2\varkappa_0^4(\varkappa_0^2-\beta^2)^4}\tag{5.4}$$

The variance for the deflection function of a shell is expressed differently depending on the value of the loading parameter β. If $\beta^2 > 2$, the coefficients γ_1 and γ_2 are real. In this case the variance is

$$c\varphi(0)=b_1+b_2\ln\frac{\gamma_1}{\gamma_2}+b_3\ln\frac{\varkappa_0}{\gamma_1}+b_4\ln\frac{\varkappa_0}{\gamma_2}\tag{5.5}$$

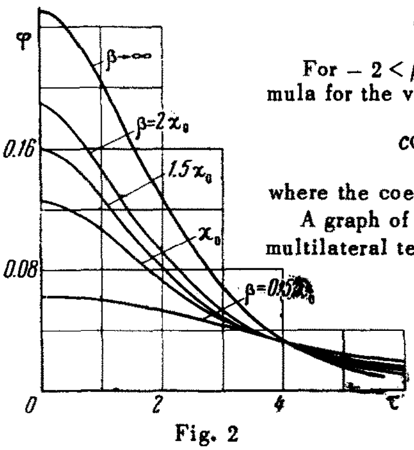

Fig. 2

For $-2<\beta^2<2$ the coefficients γ_1, γ_2 are complex. The formula for the variance will be

$$c\varphi(0)=c_1+c_2\,\mathrm{arctg}\left(\frac{2-\beta^2}{2+\beta^2}\right)^{1/2}+c_3\ln\varkappa_0\tag{5.6}$$

where the coefficients b_j and c_j depend on $\varkappa_0$ and β.

A graph of the correlation function (5.3) for a plate experiencing multilateral tension is shown in Fig. 2. The additional displacements $w_1(r)$ have a weaker correlation than the initial deviations. The correlation is magnified as the loading parameter β grows. For $\beta \to \infty$ we have $\phi(\tau) \to \phi_0(\tau)$, i.e., the initial deviations "straighten out" in the limit.

Results of computations using (5.5) and (5.6) are shown in Fig. 3. The loading parameter $\beta^2 = NR[12(1-\nu^2)]^{1/2}/Eh^2$, is plotted along the horizontal, and the dimensionless variance of the displacement $w_1(r)$ in a spherical shell along the vertical. The parameter $\varkappa_0 = k_0/k_*$ was taken equal to 0.5, 1, 2 and 4. Moreover a curve which corresponds to the limiting case of $\varkappa_0 \to \infty$ (the delta-correlated field of initial deviations) is superposed in Fig. 3. The right branches of the curve correspond to tensile stress resultants. We have $\phi(0) \to \phi_0(0)$ for $\beta \to \infty$. The left branches correspond to compressive stress resultants. As $\beta \to -2$, i.e., as the pressure tends to its critical value, determined by linear theory, the variance of the displacement $w_1(r)$ tends to infinity.

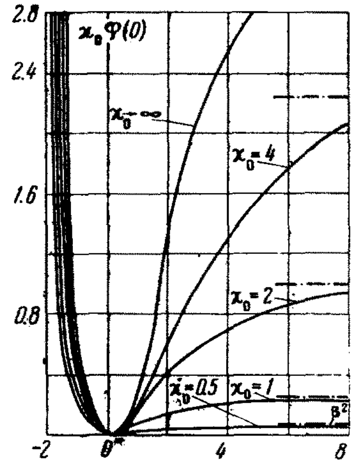

Fig. 3

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